The Law of Large Numbers I

The **Law of Large Numbers** is a fundamental concept in probability and statistics that states the average of a randomly selected sample from a large population is likely to be close to the average of the whole population. Simply, the more units of something that are measured, the closer that sample average will be to the real average of all of the units.

Stated formally the law is:

> If an event of probability *p* is observed repeatedly during independent repetitions, the ratio of the observed frequency of that event to the total number of repetitions converges towards *p* as the number of repetitions becomes arbitrarily large.

For example, the average weight of 10 M&Ms taken from a bag of M&Ms is probably closer to the "real" average weight than the average weight of 2 M&Ms taken from that same bag. This is because the sample of 10 is **larger** than the sample of 2 and better represents the whole group. If you took a sample of 25 M&Ms, the average would be even closer to the real average.

The law seems obvious. By taking multiple measurements, scientists believe they are getting a better estimate of the "right" number than with fewer measurements.

Activities:

There are two activities in the Texas Instrument website. Both are designed to run on TI graphing calculators, but they can be modified to run on Excel. To view the activities, go to "google" and type in:

law of large number ti

The site is:

http://education.ti.com/educationportal/activityexchange/activity_detail.do?cid=us&activityid=4235

In the right column to the bottom under "Other" leads to other simulations.

Other: This is Activity 1 from the EXPLORATIONS Book: Exploring Mathematics with the Probability Simulation Application.

The sum of two dice is shown in the table below:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

There are 6 x 6 = 36 combinations. Rolling a 2 occurs only 1 time and has odds of 1÷36 = 0.02777 = 2.77%. Rolling a 7 occurs 6 times or 6÷36 = 1÷6 = 0.1667 = 16.7%. The table below gives the complete probabilities.

	A	B	Probability	Percent
1	2	=1÷36	0.0385	3.85
2	3	=2÷36	0.0556	5.56
3	4	=3÷36	0.0833	8.33
4	5	=4÷36	0.1111	11.11
5	6	=5÷36	0.1389	13.89
6	7	=6÷36	0.1667	16.67
7	8	=5÷36	0.1389	13.89
8	9	=4÷36	0.1111	11.11
9	10	=3÷36	0.0833	8.33
10	11	=2÷36	0.0556	5.56
11	12	=1÷36	0.0385	3.85

To run this simulation on Excel, go to Excel.

1. Enter the following, starting at box **A1**, enter the first two columns in the table directly above. For example, A1 should have "2". B1 should have "=1÷36".
2. At the top row, select "Tools"
3. Select"Data Analysis"
4. Choose "Random Number Generator" and hit "OK".
5. Number of Varables: 6
6. Number of Random Numbers: 6
7. Distribution: Discrete
8. Value and Probability Range: Click on the red arrow in the box, and drag over the box you did in #1. It should read: A1:B11
9. Output range: Click on the red arrow and select A15, it should read: A15
10. Click "OK."

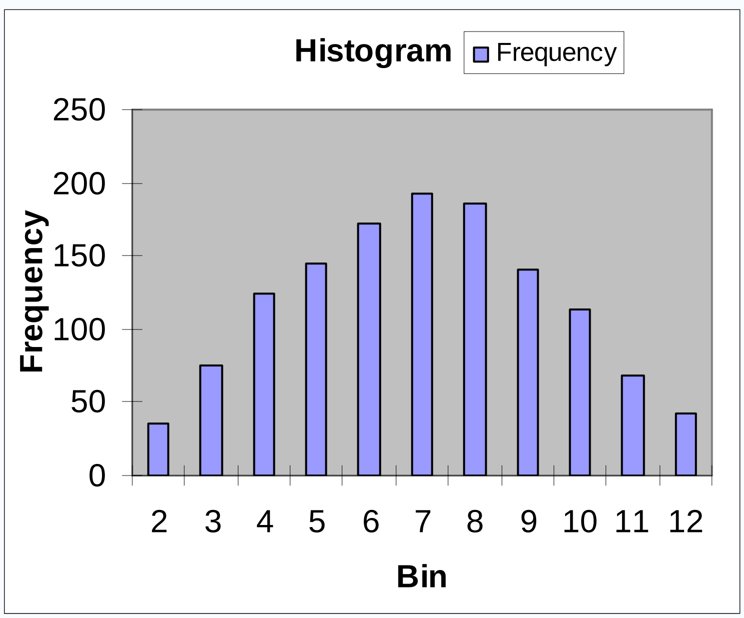
This will give you 36 readings.

Now run a Histogram to separate and select show the readings.

1. From the top, select "Tools" and "Data Analysis"
2. Choose "Histogram"
3. Input Range: Click on the red arrow and drag over the values you simulated. It should read: A15:F20
4. Bin Range: Click on the red arrow an drag over the numbers 2 to 12 in column A. It should read A1:A11
5. Output Range: Click on the red arrow and click on M1. It should read M1
6. Check off "Chart Output"
7. Click "OK."
8. Drag over the frequency numbers, N2 to N12, and copy them to column D1.

Now do 36 x 36 = 1296 simulations. In the Random Number Generator change the Number of Variables and Number of Random Numbers to 36 in the first two rows and click "OK."

Now obtain the histogram as above. The complete data set should already be selected. If not, drag over the complete data set. It should read $A$15:$AJ$50



Obtain a good histogram and copy it for your report. To get rid of the "More" column, select "Source Data" and change both endings from $13 to $12.

Now copy the frequency numbers for the 1296 run into column G1. Your Excel sheet should look like the sheet below.

A	B	C	D	E	F	G	H	I	M	N
2	0.027778		1			45			*Bin*	*Frequency*
3	0.055556		3			70			2	45
4	0.083333		3			97			3	70
5	0.111111		6			151			4	97
6	0.138889		5			179			5	151
7	0.166667		5			223			6	179
8	0.138889		5			174			7	223
9	0.111111		2			145			8	174
10	0.083333		1			104			9	145
11	0.055556		3			73			10	104
12	0.027778		2			35			11	73
									12	35
									More	0

Calculate the probabilities and compare them with the theoretical given in column B.

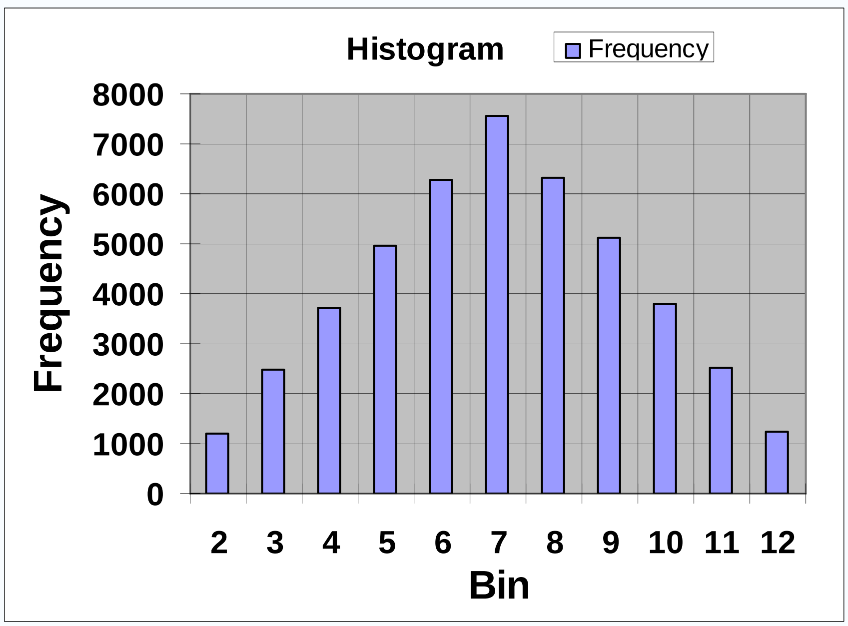
1. In column E1 type "=D1/36" and "Enter". Then click on Box E1 and go to the lower light hand corner until you see a large "**+**". Now drag it down to see the entire probabilities.
2. Do the same for column H1 except use 1296 instead of 256 (=G1/1256).

What would happen if you took 36 x 1296 = 46656 rolls? Try it. Change Number of Random variables to 1296. Run the Histogram, dragging over the complete data set. Find the probabilities dividing the frequency by 46656.

Sample results are shown on the next page.

Sample Results:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	2	0.027778		2	0.055556		36	0.028662		1214	0.02602		*Bin*	*Frequency*
2	3	0.055556		3	0.083333		75	0.059713		2490	0.053369		2	1214
3	4	0.083333		3	0.083333		125	0.099522		3711	0.07954		3	2490
4	5	0.111111		3	0.083333		145	0.115446		4968	0.106481		4	3711
5	6	0.138889		8	0.222222		172	0.136943		6263	0.134238		5	4968
6	7	0.166667		4	0.111111		192	0.152866		7554	0.161908		6	6263
7	8	0.138889		2	0.055556		186	0.148089		6318	0.135417		7	7554
8	9	0.111111		2	0.055556		141	0.112261		5126	0.109868		8	6318
9	10	0.083333		4	0.111111		113	0.089968		3819	0.081854		9	5126
10	11	0.055556		2	0.055556		68	0.05414		2515	0.053905		10	3819
11	12	0.027778		3	0.083333		43	0.034236		1238	0.026535		11	2515
		Theory			Based on			Based on			Based on		12	1238
					36			1296			46656		*More*	0



Name ________________________ Period _____ Date__________________

Law of Large Numbers I

Include the histograms and data tables in your report.

1. Compare the probabilities with the number of trials. As the number of trials gets larger from 36 to 1296 to 46656, what happens?

2. Explain how your results demononstrate the validity of the law of large numbers.

3. Compare and combine the entire classes' values for the 256 rolls. If you had 12 teams doing it, it would give you 3072 trials. Predict the outcome. Does the entire classes result agree with your predictions?

4. Consider the probabilites in these models: Lottery, Russian roulette (1 to 6 on the "wheel") and flipping 3 coins. Do you think that the Law of Large Numbers applies to these probability situations as well?

5. Why do scientists like to take as many measurements as possible?

Answer the following two extensions from the TI website.

6. Considering the steps and explorations in this activity, what is your response to the question, "How many times must an experiment involving a fair model be repeated before you can reliably determine the probability of any outcome?" That is, how large is "large" in the Law of Large Numbers?

7. Imagine that you rolled the die as many times as the cumulative number of rolls done by the class. (For example, if all 25 students roll a die 1,020 times, then the total number of rolls is 25 x 1020 = 25,500 times.) Do you think the bar graph of the data would look like the one that represented the class result? Go ahead and roll the die the same number of times as your class to verify your answer.