SRT MEMO #019

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY

WESTFORD, MASSACHUSETTS 01886

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Telephone: 978-692-4764
Fax: 781-981-0590

To: SRT Group

From: Alan E.E. Rogers

Subject: VLBI data correlation

A] Real correlation function

Memo 16 describes the format of the data files in the "VLBI" mode. The data is quantized as data files in the "VLBI" mode. The data is quantized as one bit per sample so that the "Van Vleck" correction [1] can be used to derive the continuous correlation function $R_{xy}(\tau)$ from the 1-bit or "clipped" correlation $\rho_{xy}(\tau)$ as follows:

$$R_{xy}(\tau) = \sin\left((\pi/2)\,\rho_{xy}(\tau)\right)$$

The clipped correlation function has been normalized by the number of bits correlated so that

$$\rho_{xy}(\tau) = (1/N)\sum_{t=0}^{t=N-1} x(t)\,y(t-\tau)$$

where 1 bit data from each antenna, *x(t)* and *y(t),* take on values of +1 and -1.

The 1-bit clipped data is cross-correlated using a table look-up method. The function *xcorr( )* places the data into 32 bit words. The bits are correlated using the "exclusive OR" operation and a table look-up is done on each upper and lower 16 bits. The *xinit( )* function is used to get the correlation of each 16 bit block. The results are stored in the cc[ ] array for efficient access. The lag, τ, is changed by using the shift operations.

B] Cross-spectral function

The cross-correlation is covered to a cross-spectral function $s_{xy}(w)$ given by

$$s_{xy}(w) = \sum_{\tau=-L/2+1}^{\tau=L/2} R_{xy}(\tau)\,e^{-i2\pi w\tau/M}$$

C] Complex correlation

The complex correlation (or delay function [2]) is derived using a reverse FFT transform

$$D(\tau) = \sum_{0}^{M-1} s_{xy}(w) e^{i2\pi w\tau/2M}$$

This sum can be obtained setting the negative frequencies to zero and doubling the FFT size to provide interpolation of the delay τ.

Since the sample rate is 8 Ms/s the spacing of the lags is 125 ns. The doubling of the number of points in the reverse FFT makes the spacing of the delay points 62.5 ns. The delay is determined by finding the maximum magnitude of the delay function and using parabolic interpolation between points so that

$$\tau = \tau_{\max} + \left(P_{\max+1} - P_{\max-1}\right)^2 \Big/ \left(4P_{\max} - 2P_{\max+1} - 2P_{\max-1}\right)$$

where $P = |D|^2$ and τ is in units of 62.5 ns. The interpolated maximum is

$$P = P\max + \left(P_{\max+1} - P_{\max-1}\right)^2 \Big/ \left(16P_{\max} - 8P_{\max+1} - 8P_{\max-1}\right)$$

$$P = 2P^{1/2} \big/ M$$

while the phase is $\operatorname{atan2}\left(ImD_{\max}, \operatorname{Re} D_{\max}\right)$

1] VanVleck, J.H. Middleton, D., *Proc IEEE*, **54**, 2, 1966.

2] Rogers, A.E.E., *in Methods of Experimental Physics*, **12**, part c, edited by M.L. Meeks, Academic Press, New York, 1976.

3] Thompson, A.R., Moran, J.M. and Swenson, G.W., *Interferometry and Synthesis in Radio Astronomy*, Wiley, New York, 2001.